2-1A

PRACTICE WORKSHEET – Conditional Statements

A conditional statement is a statement that can be written as an if-then statement, “if p, then q.”

The hypothesis comes after the word if.
The conclusion comes after the word then.

If you buy this cell phone, then you will receive 10 free ringtone downloads.

Sometimes it is necessary to rewrite a conditional statement so that it is in if-then form.

**Conditional:** A person who practices putting will improve her golf game.

**If-Then Form:** If a person practices putting, then she will improve her golf game.

A conditional statement has a false **truth value only** if the hypothesis (H) is true and the conclusion (C) is false.

Identify the hypothesis and conclusion of each conditional.

1. If you can see the stars, then it is night.
   
   Hypothesis: ____________________________
   
   Conclusion: ____________________________

2. If \( x \) is an even number, then \( x \) is divisible by 2.
   
   Hypothesis: ____________________________
   
   Conclusion: ____________________________

Write a conditional statement from each of the following.

3. Three noncollinear points determine a plane.

4. Congruent segments have equal measures.

5. On Tuesday, play practice is at 6:00.

6. Use the following conditional statement for Exercises 7– 8.
   
   If it is a bicycle, then it has two wheels.
   
   7. Give the hypothesis of the conditional statement. ____________________________
   
   8. Give the conclusion of the conditional statement. ____________________________
PRACTICE WORKSHEET – Conditional Statements

<table>
<thead>
<tr>
<th>Statement</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conditional</td>
<td>If a figure is a square, then it has four right angles.</td>
</tr>
<tr>
<td>Converse: Switch H and C.</td>
<td>If a figure has four right angles, then it is a square.</td>
</tr>
</tbody>
</table>

Show that each conditional is false by finding a counterexample.

1. If it is 12:00 noon, then the sun is shining.
2. If a number is divisible by 3, then it is odd.

Write the converse of each conditional.

3. If you drink milk, then you will be strong.
4. If a rectangle has four sides the same length, then it is a square.
5. If a rectangle has four sides the same length, then it is a square.
6. If you do not sleep, you will be tired.

Write the converse and decide whether the converse is true or false. If the converse is false, give a counterexample.

7. If the sun is shining, then it is 12:00 noon.
8. If the number is divisible by 3, then the number is odd.
9. If an angle is 90°, then it is a right angle.
A biconditional statement combines a conditional statement, “if $p$, then $q$,” with its converse, “if $q$, then $p$.”

**Conditional:** If the sides of a triangle are congruent, then the angles are congruent.

**Converse:** If the angles of a triangle are congruent, then the sides are congruent.

**Biconditional:** The sides of a triangle are congruent if and only if the angles are congruent.

...where $p = \text{hypothesis}$ and $q = \text{conclusion}$

**For each conditional, write the converse and a biconditional statement.**

1. **Conditional:** If the date is July 4th, then it is Independence Day.
   - Converse: ________________________
   - Biconditional: ____________________

2. **Conditional:** If a figure has 10 sides, then it is a decagon.
   - Converse: ________________________
   - Biconditional: ____________________

**Write each definition as a biconditional.**

3. An isosceles triangle has at least two congruent sides.
   - ________________________
   - ________________________

4. A cube is a three-dimensional solid with six square faces.
   - ________________________
   - ________________________
1. A biconditional statement combines a conditional and its __________________________.

2. A biconditional statement can be written in the form “p if and only if q,” which means “if p, then q, and if ________, then ________.”

Write the converse from each given biconditional.

3. Biconditional: A cat is happy if and only if it is purring.
   Conditional: If a cat is happy, then it is purring.
   Converse: ____________________________________________________________

4. Biconditional: A figure is a segment if and only if it is straight and has two endpoints.
   Conditional: If a figure is a segment, then it is straight and has two endpoints.
   Converse: ____________________________________________________________

Write a biconditional from each given conditional and converse.

5. Conditional: If two angles share a side, then they are adjacent.
   Converse: If two angles are adjacent, then they share a side.
   Biconditional: ________________________________________________________

6. Conditional: If your temperature is normal, then your temperature is 98.6°F.
   Converse: If your temperature is 98.6°F, then your temperature is normal.
   Biconditional: ________________________________________________________

Write the conditional statement and converse within each biconditional.

7. The tea kettle is whistling if and only if the water is boiling.
   Conditional: __________________________________________________________
   Converse: ____________________________________________________________

Some figures that are *piggles* are shown below, as are some *nonpiggles*.

![Piggles and Nonpiggles](image)

8. Definition of *piggle*: _________________________________________________

Tell whether each of the following is a *piggle*.

9.  
10.  
11.  
With inductive reasoning, you use examples to make a conjecture. With deductive reasoning, you use facts, definitions, and properties to draw conclusions and prove that conjectures are true.

One form of deductive reasoning that draws conclusions from a true conditional \( p \rightarrow q \) and a true statement \( p \) is called the **Law of Detachment**.

<table>
<thead>
<tr>
<th>Law of Detachment</th>
<th>If ( p \rightarrow q ) is true and ( p ) is true, then ( q ) is true.</th>
</tr>
</thead>
</table>

- Tom knows that if he misses the practice the day before a game, then he will not be a starting player in the game.
- Tom misses practice on Tuesday.
- **Conclusion:** He will not be able to start in the game on Wednesday.

Another way to make a valid conclusion is to use the **Law of Syllogism**.

<table>
<thead>
<tr>
<th>Law of Syllogism</th>
<th>If ( p \rightarrow q ) is true and ( q \rightarrow r ) is true, then ( p \rightarrow r ) is also true.</th>
</tr>
</thead>
</table>

- **Given:** If you have a horse, then you have to feed it. If you have to feed a horse, then you have to get up early every morning.
- **Conclusion:** If you have a horse, then you have to get up early every morning.

*Determine if a valid conclusion can be reached from the two true statements using the Law of Detachment or the Law of Syllogism. If a valid conclusion is possible, state it and the law that is used. If a valid conclusion does not follow, write no valid conclusion.*

1. If Jim is a Texan, then he is an American.
   Jim is a Texan.

2. If Spot is a dog, then he has four legs.
   Spot has four legs.

3. If Rachel lives in Tampa, then Rachel lives in Florida.
   If Rachel lives in Florida, then Rachel lives in the United States.

4. If October 12 is a Monday, then October 13 is a Tuesday.
   October 12 is a Monday.

5. If Henry studies his algebra, then he passes the test.
   If Henry passes the test, then he will get a good grade.
Use the Law of Detachment to draw a conclusion.

1. If the football team wins on Friday night, then practice is canceled for Monday.
   The football team won by 7 points on Friday night.

2. If a triangle has one 90° angle, then the triangle is a right triangle.
   In \(\triangle DEF\), \(m\angle E = 90\).

Use the Law of Syllogism to draw a conclusion.

3. If two lines are not parallel, then they intersect.
   If two lines intersect, then they intersect at a point.

4. If you vacation at the beach, then you must like the ocean.
   If you like the ocean, then you will like Florida.

If possible, use the Law of Detachment to draw a conclusion. If not possible, write *not possible*.

5. If a person lives in Omaha, then he or she lives in Nebraska.
   Tamika lives in Omaha.

6. If Robbie wants to save money to buy a car, he must get a part-time job.
   Robbie started a new job yesterday at a grocery store.

Use the Law of Detachment and the Law of Syllogism to draw conclusions from the following statements.

7. If it is raining, the temperature is greater than 32°F.
   If the temperature is greater than 32°F, then it is not freezing outside.
   It is raining.

8. If you live in Providence, then you live in Rhode Island.
   If you live in Rhode Island, then you live in the smallest state in the United States.
   Shannon lives in Providence.
2-4A  PRACTICE WORKSHEET – Reasoning in Algebra

A proof is a logical argument that shows a conclusion is true. An algebraic proof uses algebraic properties, including the Distributive Property and the properties of equality.

<table>
<thead>
<tr>
<th>Properties of Equality</th>
<th>Symbols</th>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>Addition</td>
<td>If ( a = b ), then ( a + c = b + c ).</td>
<td>If ( x = -4 ), then ( x + 4 = -4 + 4 ).</td>
</tr>
<tr>
<td>Subtraction</td>
<td>If ( a = b ), then ( a - c = b - c ).</td>
<td>If ( r + 1 = 7 ), then ( r + 1 - 1 = 7 - 1 ).</td>
</tr>
<tr>
<td>Multiplication</td>
<td>If ( a = b ), then ( ac = bc ).</td>
<td>If ( \frac{k}{2} = 8 ), then ( \frac{k}{2}(2) = 8(2) ).</td>
</tr>
<tr>
<td>Division</td>
<td>If ( a = 2 ) and ( c \neq 0 ), then ( \frac{a}{c} = \frac{b}{c} ).</td>
<td>If ( 6 = 3t ), then ( \frac{6}{3} = \frac{3t}{3} ).</td>
</tr>
<tr>
<td>Reflexive</td>
<td>( a = a )</td>
<td>15 = 15</td>
</tr>
<tr>
<td>Symmetric</td>
<td>If ( a = b ), then ( b = a ).</td>
<td>If ( n = 2 ), then ( 2 = n ).</td>
</tr>
<tr>
<td>Transitive</td>
<td>If ( a = b ) and ( b = c ), then ( a = c ).</td>
<td>If ( y = 3^2 ) and ( 3^2 = 9 ), then ( y = 9 ).</td>
</tr>
<tr>
<td>Substitution</td>
<td>If ( a = b ), then ( b ) can be substituted for ( a ) in any expression.</td>
<td>If ( x = 7 ), then ( 2x = 2(7) ).</td>
</tr>
</tbody>
</table>

For Exercises 1–12, write the letter of each property next to its definition. The letters \( a \), \( b \), and \( c \) represent real numbers.

1. If \( a = b \), then \( b = a \). \( \text{A. Addition Property of Equality} \)
2. If \( a = b \), then \( ac = bc \). \( \text{B. Subtraction Property of Equality} \)
3. \( AB \cong AB \) \( \text{C. Multiplication Property of Equality} \)
4. \( a = a \) \( \text{D. Division Property of Equality} \)
5. If \( a = b \), then \( a + c = b + c \). \( \text{E. Reflexive Property of Equality} \)
6. \( a(b + c) = ab + ac \) \( \text{F. Symmetric Property of Equality} \)
7. If \( a = b \) and \( b = c \), then \( a = c \). \( \text{G. Transitive Property of Equality} \)
8. If \( \angle P = \angle Q \), then \( \angle Q \cong \angle P \). \( \text{H. Substitution Property of Equality} \)
9. If \( \angle A \cong \angle B \) and \( \angle B \cong \angle C \), \( \text{I. Distributive Property} \)
   then \( \angle A \cong \angle C \). \( \text{J. Reflexive Property of Congruence} \)
10. If \( a = b \) and \( c \neq 0 \), then \( \frac{a}{c} = \frac{b}{c} \). \( \text{K. Symmetric Property of Congruence} \)
11. If \( a = b \), then \( b \) can be substituted for \( a \) in any expression. \( \text{L. Transitive Property of Congruence} \)

Write a justification for each step.

13. \( \frac{7x - 3}{HJ} = \frac{HI + IJ}{3x - 3} \)
   \( HJ = HI + IJ \)
   \( 7x - 3 = (2x + 6) + (3x - 3) \)
   \( 7x - 3 = 5x + 3 \)
   \( 7x = 5x + 6 \)
   \( 2x = 6 \)
   \( x = 3 \)

When solving an algebraic equation, justify each step by using a definition property, or piece of given information.

| Given: \( 2(x+3)=12 \)  | 1. Given |
| Prove: \( x=3 \)  | 2. Distribution |
| Seg. Add. Post.  | 3. Add/Sub Property of Equality |

<table>
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<tr>
<th>Reasons</th>
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<tbody>
<tr>
<td>1. Given</td>
</tr>
<tr>
<td>2. Distribution</td>
</tr>
<tr>
<td>3. Add/Sub Property of Equality</td>
</tr>
<tr>
<td>4. Mult/Div Property of Equality</td>
</tr>
</tbody>
</table>
PRACTICE WORKSHEET – Reasoning in Algebra

Properties of Congruence

Reflexive Property \( \overline{AB} \cong \overline{AB} \)
\( \angle A \cong \angle A \)

Symmetric Property
If \( \overline{AB} \cong \overline{CD} \), then \( \overline{CD} \cong \overline{AB} \)
If \( \angle A \cong \angle B \), then \( \angle B \cong \angle A \)

Transitive Property
If \( \overline{AB} \cong \overline{CD} \) and \( \overline{CD} \cong \overline{EF} \), then \( \overline{AB} \cong \overline{EF} \)
If \( \angle A \cong \angle B \) and \( \angle B \cong \angle C \), then \( \angle A \cong \angle C \)

Use the given property to complete each statement.

1. Symmetric Property of Equality
   If \( MN = UT \), then ________________________________.

2. Transitive Property of Equality
   If \( SB = VT \) and \( VT = MN \), then ________________________________.

3. Reflexive Property of Congruence
   \( \overline{LL} \cong \) ________________________________.

Give a reason for each step.

4. \( 0.25x + 2x + 12 = 39 \)
   \( 2.25x + 12 = 39 \) ________________________________
   \( 2.25x = 27 \) ________________________________
   \( 225x = 2700 \) ________________________________
   \( x = 12 \) ________________________________

Name the property that justifies each statement.

5. If \( m \angle G = 35 \) and \( m \angle S = 35 \), then \( m \angle G = m \angle S \) ________________________________.

6. If \( 10x + 6y = 14 \) and \( x = 2y \), then \( 10(2y) + 6y = 14 \) ________________________________.

7. If \( \overline{JK} \cong \overline{LM} \), then \( \overline{LM} \cong \overline{JK} \) ________________________________.

Give a reason for each step.

8. Prove that if \( 2(x - 3) = 8 \), then \( x = 7 \).
   Given: \( 2(x - 3) = 8 \)
   Prove: \( x = 7 \)

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. ( 2(x - 3) = 8 )</td>
<td>a.</td>
</tr>
<tr>
<td>b. ( 2x - 6 = 8 )</td>
<td>b.</td>
</tr>
<tr>
<td>c. ( 2x = 14 )</td>
<td>c.</td>
</tr>
<tr>
<td>d. ( x = 7 )</td>
<td>d.</td>
</tr>
</tbody>
</table>
2-5A  PRACTICE WORKSHEET – Proving Angles Congruent

Theorem 2-1 - Vertical Angles Congruence Theorem
Vertical angles are congruent
\[ \angle 1 \equiv \angle 3 \text{ and } \angle 2 \equiv \angle 4 \]

Find the values of the variables.
1. \[ (3x - 40)° \quad (2x - 10)° \]
   \[ x = \underline{\hspace{2cm}} \]

2. \[ (x + 10)° \quad (4x - 35)° \]
   \[ x = \underline{\hspace{2cm}} \]

Theorem 2-2 - Congruent Supplements Theorem
If two angles are supplementary to the same angle (or to congruent angles), then they are congruent.
If \( \angle 1 \) and \( \angle 2 \) are supplementary and \( \angle 2 \) and \( \angle 3 \) are supplementary, then \( \angle 1 \equiv \angle 3 \)

Postulate - Linear Pair Postulate
If two angles form a linear pair, then they are supplementary.
\( \angle 1 \) and \( \angle 2 \) are supplementary

Find the values of the variables.
3. \[ (6y - 10)° \quad (6y + 10)° \]
   \[ y = \underline{\hspace{2cm}} \]

4. \[ x° \quad (3x + 20)° \]
   \[ x = \underline{\hspace{2cm}} \]

5. \( \angle A \) is three times as large as its supplement, \( \angle B \).
   What are the measures of \( \angle A \) and \( \angle B \)?
   \( \angle A = \underline{\hspace{2cm}} \quad \angle B = \underline{\hspace{2cm}} \)
Find the values of the variables.

1. \[ 32^\circ \]

\[ (9x + 4)^\circ \]

\[ x = \phantom{0} \]

2. \[ \angle A \]

\[ \angle B \]

\[ z = \phantom{0} \]

Find the values of the variables.

3. \[ 3x^\circ \]

\[ 75^\circ \]

\[ \frac{1}{8} \]

\[ x = \phantom{0} \]

\[ y = \phantom{0} \]

4. \[ (3x + 8)^\circ \]

\[ (5x - 20)^\circ \]

\[ (5x + 4y)^\circ \]

\[ x = \phantom{0} \]

\[ y = \phantom{0} \]

5. \[ (7x + 3)^\circ \]

\[ (4x + 1)^\circ \]

\[ 65^\circ \]

\[ x = \phantom{0} \]

6. \[ \angle A \]

\[ \angle B \]

\[ \angle A \text{ is one-eighth } \left( \frac{1}{8} \right) \text{ times as large as its complement, } \angle B. \]

What are the measures of \( \angle A \) and \( \angle B \)?

\[ \angle A = \phantom{0} \]

\[ \angle B = \phantom{0} \]