

9-5: Rational Equations

* Sometimes we need to solve equations that have fractional form.

To do so, it is wise to eliminate the denominators by multiplying all terms by the LCD.

(Ex 1) $\frac{2}{(y+4)} + \frac{y}{10} = \frac{1}{5}$ Solve for y !

LCD is $5(y+4)$

$$\frac{2}{\cancel{(y+4)}} \cdot 5(\cancel{y+4}) + \frac{y}{1} \cdot 5(\cancel{y+4}) = \frac{1}{5} \cdot 5(\cancel{y+4})$$

$$\begin{array}{r} 10 + 5y^2 + 20y = y + 4 \\ -4 \qquad \qquad \qquad -y \qquad -4 \end{array}$$

$$6 + 5y^2 + 19y = 0$$

$$5y^2 + 19y + 6 = 0$$

Doesn't factor.
use Quadratic
Formula

~~$(5y \quad)(y \quad) = 0$~~

$$5y^2 + 19y + 6 = 0$$

$$a = 5, \quad b = 19, \quad c = 6$$

$$X = \frac{-19 \pm \sqrt{19^2 - 4(5)(6)}}{10}$$

$$X = \frac{-19 \pm \sqrt{361 - 120}}{10}$$

$$X = \frac{-19 \pm \sqrt{241}}{10}$$

$$X = \frac{-19 \pm 15.5}{10}$$

$$X = -0.35 \text{ or } -3.45$$

$$\text{Ex 2} \quad \frac{x+1}{3(x-2)} = \frac{5x}{6} + \frac{1}{x-2}$$

$$\text{LCD: } 6(x-2)$$

$$\frac{(x+1) \cdot \cancel{6(x-2)}}{\cancel{3(x-2)}} = \frac{5x}{6} \cdot \cancel{6(x-2)} + \frac{1}{\cancel{x-2}} \cdot \cancel{6(x-2)}$$

$$\begin{array}{r} 2x+2 \\ -2x-2 \\ \hline \end{array} = \begin{array}{r} 5x^2-10x+6 \\ -2x-2 \\ \hline \end{array}$$

$$0 = 5x^2 - 12x + 4$$

$$0 = (5x-2)(x-2)$$

$$5x-2=0$$

$$x = \frac{2}{5}$$

and

$$x-2=0$$

$$\cancel{x=2}$$

Can't have division
by zero.

$$\text{Ex 3 } \frac{1}{(m-4)} = \frac{2}{(m-2)}$$

$$\text{LCD: } (m-4)(m-2)$$

$$\frac{1}{\cancel{m-4}} \cdot (\cancel{m-4})(m-2) = \frac{2}{\cancel{m-2}} \cdot (m-4)(\cancel{m-2})$$

$$\begin{array}{r} m-2 = 2m-8 \\ -m \qquad -m \end{array}$$

$$\begin{array}{r} -2 = m - 8 \\ +8 \qquad +8 \end{array}$$

$$\boxed{6 = m}$$

Check by substituting
6 for m

$$\frac{1}{2} = \frac{2}{4}$$

✓ It works!

Ex 4. $\frac{r+2}{2r+1} = \frac{r}{3} + \frac{3}{4r+2}$
 $\downarrow 2(2r+1)$

LCM: $6(2r+1)$

$$\frac{(r+2) \cdot 6(2r+1)}{2r+1} = \frac{r}{3} \cdot \frac{6(2r+1)}{2r+1} + \frac{3}{4r+2} \cdot \frac{6(2r+1)}{2(2r+1)}$$

$$6r + 12 = 4r^2 + 2r + 9$$

$$-6r = 12 \qquad \qquad \qquad -6r \quad -12$$

$$0 = 4r^2 - 4r - 3$$

$$0 = (2r - 3)(2r + 1)$$

$$r = \frac{3}{2} \text{ and } \frac{-1}{2}$$

Not a valid solution, division by zero

* must always check for excluded solutions *

9-5: Continued

EX 5 $\frac{3m+2}{5m} + \frac{2m-1}{2m} = 4$

LCD: $10m$

$$\frac{(3m+2)(\cancel{10m}^2)}{\cancel{5m}} + \frac{(2m-1)(\cancel{10m}^5)}{\cancel{2m}} = 4(10m)$$

$$6m + 4 + 10m - 5 = 40m$$

$$\begin{array}{r} 16m - 1 = 40m \\ -16m \quad \quad -16m \\ \hline \end{array}$$

$$\frac{-1}{24} = \frac{\cancel{24m}}{\cancel{24}}$$

$m = -\frac{1}{24}$

Ex 6. $b^2 + \frac{17b}{6} = \frac{1}{2}$

LCD: 6 multiply through

$$b^2(6) + \frac{17b(6)}{6} = \frac{1}{2}(6)$$

$$6b^2 + 17b = \frac{3}{-3}$$

$$6b^2 + 17b - 3 = 0$$

~~$$(3b)(2b) = 0$$~~

$$(6b - 1)(b + 3) = 0$$

$$\begin{array}{r} 6b - 1 = 0 \\ +1 \quad +1 \\ \hline \frac{6b}{6} = \frac{1}{6} \\ b = \frac{1}{6} \end{array}$$

$$b + 3 = 0$$

$$b = -3$$

Ex 7 The ratio of 5 more than
 a number to 7 less than
 a number is 7:1
 What is the number?

$$\cancel{(x-7)} \frac{x+5}{\cancel{x-7}} = \frac{7(x-7)}{1} \quad \text{LCD} = x-7$$

$$\begin{array}{r} x+5 = 7x-49 \\ -x \qquad -x \end{array}$$

$$\begin{array}{r} 5 = 6x - 49 \\ +49 \qquad +49 \end{array}$$

$$\frac{54}{6} = \frac{6x}{6}$$

$$x = 9$$

$$\frac{14}{2} = \frac{7}{1}$$

P. 582, # 19-30, 37