Notes 8-8 Continued

Ex 1.) Determine if $f(x) = 3x - 4$ and $g(x) = \frac{x+4}{3}$ are inverse functions.

$$F[G(x)] = 3\left(\frac{x+4}{3}\right) - 4$$

$$x + 4 - 4 = \boxed{X}$$

$$G[f(x)] = \frac{(3x-4)+4}{3} = \frac{3x}{3} = \boxed{X}$$

Since they both equal X, they're inverse functions.

$$F(8) = 3(8) - 4 = 20$$

(8: input; 20: output)

$$G(20) = \frac{20+4}{3} = \frac{24}{3} = 8$$

(20: input now)

Example 2

Find the inverse of $f(x) = 3x + 6$

$y = 3x + 6$

Solve for x

$$y = 3x + 6$$
$$-6 \quad\quad -6$$

$$\frac{y-6}{3} = \frac{3x}{3}$$

$$x = \frac{y-6}{3}$$

But, we have a problem.

x always need to be the input, y is your output.

So, Switch them: $y = \frac{x-6}{3}$

* Easier method to find inverses → swap x & y First, then solve.

$$F(x) = 3x + 6$$

$$y = 3x + 6$$

Swap

$$x = 3y + 6$$
$$-6 \quad -6$$

solve for y

$$\frac{x-6}{3} = \frac{3y}{3} \rightarrow \text{so } y = \frac{x-6}{3}$$

Since this is the inverse of $F(x)$, we call it $F^{-1}(x) = \frac{x-6}{3}$

Prove we're correct

$$F[F^{-1}(x)] = 3\left(\frac{x-6}{3}\right) + 6$$

$$x - 6 + 6 = (x)$$

$$F^{-1}[F(x)] = \frac{(3x+6) - 6}{3}$$

$$= \frac{3x}{3} = (x)$$

proof

p. 532

(20) $F(x) = 4x$ Find $F^{-1}(x)$ and prove they're inverse functions.

Same as $y = 4x$

Swap x and y

So... $x = 4y$ always solve for y.

$$\frac{x}{4} = \frac{4y}{4}$$

$$y = \frac{x}{4}$$

This is the inverse of $F(x)$,

$$F^{-1}(x) = \frac{x}{4}$$

$$F[F^{-1}(x)] = 4\left(\frac{x}{4}\right) = x$$

$$F^{-1}[F(x)] = \frac{(4x)}{4} = x$$

Example 3

Find inverse of $F(x) = x^2 + 4$.

Determine if this $F^{-1}(x)$ is still a function.

same as $y = x^2 + 4$

Swap x & y $\rightarrow$ $x = y^2 + 4$

Solve for y $\rightarrow$ $x = y^2 + 4$

$-4 \quad\quad -4$

$\sqrt{x-4} = \sqrt{y^2}$

$\pm\sqrt{x-4} = y$ ★

$F^{-1}(x) = \pm\sqrt{x-4}$

Inverse ↗ but NOT a function because there are two y-values for every x value