Notes 8-5: Rational Zero Theorem

p. 509 For any polynomial function,
Let "p" be a factor of the last term's coefficient and let "q" be a factor of the leading (first) term's coefficient.
Thus, the possibilities of $\frac{p}{q}$ are possible rational <u>roots</u>.

example $p(x) = x^3 - 3x - 2$

Degree: 3, three complex roots

positive roots: 1

negative roots: $-x^3 + 3x - 2$

2 or 0

imaginary roots: 2 or 0

* Rational Zero Theorem

$p(x) = x^3 - 3x - 2$

$p: -2 \rightarrow \pm 1, \pm 2$

$q: 1 \rightarrow \pm 1$

$\frac{p}{q}: 1, -1, 2, -2$

$$
\begin{array}{r|rrr|r}
2 & 1 & 0 & -3 & -2 \\
 & & 2 & 4 & 2 \\
\hline
 & 1 & 2 & 1 & 0
\end{array}
$$

$$x^2 + 2x + 1 = 0$$

$$(x+1)(x+1) = 0$$

$$x = -1, -1, 2$$

Example: $h(x) = 6x^3 + 11x^2 - 3x - 2$

Degee: 3

positive: 1

$h(-x)$ negative: $-6x^3 + 11x^2 + 3x - 2$ (2 or 0)

imaginary: 2 or 0

p: $-2 \rightarrow \pm 1, \pm 2$

q: $-6 \rightarrow \pm 1, \pm 6, \pm 2, \pm 3$

$\frac{p}{q}$: $1, -1, \frac{1}{6}, -\frac{1}{6}, \frac{1}{2}, -\frac{1}{2}, \frac{1}{3}, -\frac{1}{3}$

$2, -2, \frac{2}{3}, -\frac{2}{3}$

2	6	11	-3	-2
		12	46	86
	6	23	43	R 84

$$
\begin{array}{r|rrr|r}
1 & 6 & 11 & -3 & -2 \\
 & & 6 & 17 & 14 \\
\hline
 & 6 & 17 & 14 & R\,12
\end{array}
$$

$$
\begin{array}{r|rrr|r}
-2 & 6 & 11 & -3 & -2 \\
 & & -12 & 2 & 2 \\
\hline
 & 6 & -1 & -1 & 0
\end{array}
$$

one root is $x = -2$

$$6x^2 - x - 1 = 0$$

$$(3x + 1)(2x - 1) = 0$$

$$3x + 1 = 0 \qquad x = -\tfrac{1}{3}$$

$$2x - 1 = 0 \qquad x = \tfrac{1}{2}$$