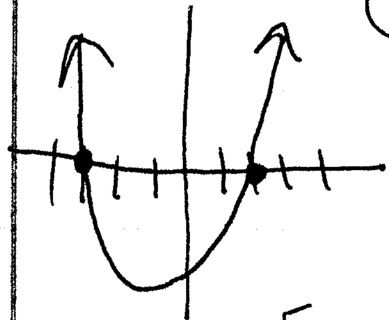
Notes on 8-4: Roots & Zeroes

Graph: $x^2 + x - 6 = 0$

$(x + 3)(x - 2) = 0$

$x = -3$ or 2



Fundamental Theorem of Algebra: Every polynomial equation with degree greater than zero has at least one root in the set of complex numbers.

Recall: Complex numbers are of the form $a + bi$. They include real and imaginary #'s.

Corollary: A polynomial equation of the form $P(x) = 0$ of degree n with complex coefficients has exactly n roots in the set of complex numbers.

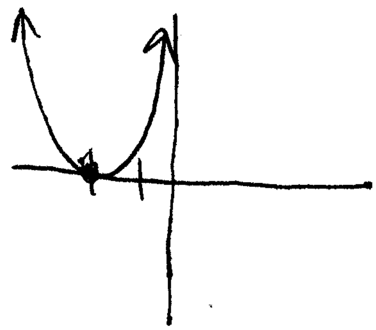
$x^2 + 4x + 4 = 0$

* must have 2 complex roots.

$(x + 2)(x + 2) = 0$

$x = -2$ and -2

* This is called a "Double root"



$x^3 + x = 0$

* we expect 3 roots *

$x(x^2 + 1) = 0$

$x = 0$

$x^2 + 1 = 0$

Same as $x^2 + 0x + 1$

$a = 1,\ b = 0,\ c = 1$

$$\frac{\pm\sqrt{-4(1)(1)}}{2}$$

$$\frac{\pm\sqrt{-4}}{2} = \frac{\pm 2i}{2}$$

$1i$ or $-1i$

Ex. Find all roots of

$$x^3 + 3x^2 - 10x - 24$$

when one root is $x = -4$

$$(x+4) = 0$$

-4	1	3	-10	-24
$+$		-4	$+4$	24
	1	-1	-6	Remainder $\emptyset$

$$x^2 - x - 6 = 0$$

$$(x - 3)(x + 2) = 0$$

$$x = 3, -2, -4$$

Complex Conjugate Theorem:

Suppose a & b are real numbers with $b \neq 0$. If $a + bi$ is a root of a polynomial function, then $a - bi$ is also a root!

Find all roots (Zeroes) of

$F(x) = x^3 - 5x^2 - 7x + 51$

if $4-i$ is a zero.

* we know its conjugate is $4+i$

$x = 4-i$
$x = 4+i$

$[x-(4-i)][x-(4+i)] = 0$

F O I L

$x^2 - (4+i)x - (4-i)x + (4-i)(4+i)$

$x^2 - 4x - ix - 4x + ix + 16 - i^2$

$\qquad\qquad\qquad\qquad\qquad\qquad - (-1)$

$x^2 - 8x + 16 + 1$

$(x^2 - 8x + 17)(\ ?\ )$

Must try Long Division

$$
\begin{array}{r}
x + 3 \\
x^2 - 8x + 17 \overline{\big) \, x^3 - 5x^2 - 7x + 51} \\
-(x^3 - 8x^2 + 17x) \\
\hline
3x^2 - 24x + 51 \\
-(3x^2 - 24x + 51) \\
\hline
\end{array}
$$

$x + 3 = 0$

$x = -3$

$4 + i$

$4 - i$

$f(x) = x^3 + 7x^2 + 25x + 175$

one root is $5i$, Find all of 'em!

$$\begin{array}{r} x = 5i \\ -5i \quad -5i \\ \hline x - 5i = 0 \end{array} \qquad \begin{array}{r} x = -5i \\ +5i \quad +5i \\ \hline x + 5i = 0 \end{array}$$

$$(x - 5i)(x + 5i)$$

$$x^2 + 5ix - 5ix - 25i^2$$

(-1)

$$x^2 + 25$$

$$\begin{array}{r} x + 7 \\ x^2 + 0x + 25 \overline{\big) x^3 + 7x^2 + 25x + 175} \\ x^3 + 0x^2 + 25x \\ \hline 7x^2 + 175 \\ -(7x^2 + 175) \\ \hline 0 \end{array}$$

$x = -7, 5i, -5i$