Sec 11-5: Infinite Geometric Series

We're going to look at three different series, all starting with $\frac{1}{2}$, but with different values of r (multiplier). Find the sum to six terms.

a.) $r = 5$; $\frac{1}{2} + \frac{5}{2} + \frac{25}{2} + \frac{125}{2} + \frac{625}{2} + \frac{3125}{2}$

$$S_6 = \frac{3906}{2} = \boxed{1953}$$

* every # we add gets much bigger.

b.) $r = \frac{1}{2}$; $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \frac{1}{64}$

$.5 \quad .25 \quad .125 + .0625 + .03125 + .015625$

$$S_6 = \frac{63}{64} = .984375 \approx 1$$

* every # we add gets smaller & smaller.

when values of r are less than one whole, the sum approaches a limit because the numbers we keep adding get so small they don't really effect the total.

This is called an Infinite Geometric Series!

$\boxed{\text{Def:}}$ The sum, S, of an Infinite Geometric Series where $-1 < r < 1$ is given by:

$$\boxed{S = \frac{a_1}{1 - r}}$$

* To compute these, must have a_1 and r.

__But__, r must be less than one whole (positive or negative.)

Ex 1 $36 + 24 + 16 + \ldots$

Is this an in~~fi~~finite Geometric Series?

Check value of r: $\frac{2}{3}$ ✓

Find Sum: $S = \frac{a_1}{1-r}$

$$S = \frac{36}{1-\frac{2}{3}} \rightarrow \frac{36}{\left(\frac{1}{3}\right)} = \boxed{108}$$

Ex 2 $a_1 = 6$, $r = -\frac{1}{3}$

$$S = \frac{6}{1 - -\frac{1}{3}} \rightarrow \frac{6}{\left(\frac{4}{3}\right)} = \boxed{4.5}$$

EX 3 $16 - 24 + 36 + \ldots$

Find $r = -1.5 \leftarrow$ Not Less than 1.

does not exist

Ex 4

Infinity

$$\sum_{n=1}^{\infty} 40\left(\frac{3}{5}\right)^{n-1}$$

Find a_1: $40\left(\frac{3}{5}\right)^{1-1} \rightarrow 40\left(\frac{3}{5}\right)^{0} \rightarrow 40$

a_2: $40\left(\frac{3}{5}\right)^{2-1} \rightarrow 40\left(\frac{3}{5}\right)^{1} \rightarrow 24$

Find $r = \frac{24}{40} = 0.6$ ✓

$$S = \frac{40}{1-0.6} \rightarrow \frac{40}{0.4} = \boxed{100}$$

p. 680 # 17-31 ODD