11-4: Geometric Series

A Geometric Series is the sum of the terms of a Geometric Sequence.

A parental Family tree is a type of Geometric Series.

Parents	Grandparents	Great-gp	GGGp
2	4	8	16

$$S_4 = 2 + 4 + 8 + 16 = 30$$

We need a Formula for large values of n. Let's Explore!!!!!

$$S_4 = 2 + 4 + 8 + 16$$

$$-2S_4 = \quad 4 + 8 + 16 + 32$$

$$S_4 - 2S_4 = 2 + 0 + 0 + 0 - 32$$

$$\frac{S_4(1-2)}{(1-2)} = \boxed{\frac{2-32}{1-2}} = 30$$

$S_4 = 2 + 4 + 8 + 16$

Analyze: a_1, $a_1 r^n$

$$S_4 = \frac{2 - 32}{1 - 2}$$

$1 - 2 \leftarrow r$

$\rightarrow 1$ Constant

So $\boxed{S_n = \dfrac{a_1 - a_1 r^n}{1 - r}} \quad r \neq 1$

Ex1 Sum of First 6 terms

$a_1 = 5, \; r = -2$

$$S_6 = \frac{5 - 5(-2)^6}{1 - -2}$$

$$= \frac{5 - 5(64)}{3} = \frac{5 - 320}{3}$$

$$= \frac{-315}{3}$$

$$\boxed{S_6 = -105}$$

Ex 2 $a_1 = 5, \; r = 2, \; n = 14$

$$S_{14} = \frac{5 - 5(2)^{14}}{1 - 2}$$

$$= \frac{5 - 5(16{,}384)}{-1} = \frac{5 - 81{,}920}{-1}$$

$$= \frac{-81{,}915}{-1}$$

$$\boxed{S_{14} = 81{,}915}$$

Sometimes we know the last term, a_n, but not how many terms in the series.

Alternate Geo series:

$$\boxed{S_n = \frac{a_1 - a_n r}{1 - r}}$$

Ex3 Find S_n when $a_1 = 729$, $a_n = -3$, $r = -\frac{1}{3}$

Alt. $S_n = \dfrac{a_1 - a_n \cdot r}{1 - r}$

$$S_n = \frac{729 - (-3)\left(-\frac{1}{3}\right)}{1 - -\frac{1}{3}}$$

$$= \frac{729 - 1}{\frac{4}{3}} = \frac{728}{\frac{4}{3}}$$

$728 \div \frac{4}{3} \rightarrow \frac{728}{1} * \frac{3}{4} = \boxed{546}$

Ex4 Find a_1 when $S_7 = 70,933$; $r = 4$

$$S_7 = \frac{a_1 - a_1 r^n}{1 - r}$$

$$70,933 = \frac{a_1 - a_1(4)^7}{1-4}$$

$$(-3)70,933 = \frac{a_1 - a_1(16,384)}{-3}(-3)$$

$$-212799 = 1a_1 - a_1(16,384)$$

$$\frac{-212799}{(1-16,384)} = \frac{a_1(1-16,384)}{(1-16,384)}$$

$$12.989 = a_1$$

Ex 5 $\sum_{n=1}^{5} 4(3)^{n-1}$ Find Sum

$4(3)^{1-1}$ $4(3)^{2-1}$

$4(3)^0$ $4(3)$

4 12

$r = 3$ $a_1 = 4$ $n = 5$

$$S_5 = \frac{4 - 4(3)^5}{1-3}$$

$$= \frac{4 - 972}{-2} = 484$$

Ex 6 Find sum: $a_1 = 12, r = -3, a_5 = 972$

$n = 5, \; a_1 = 12, \; r = -3$

$$S_5 = \frac{12 - 12(-3)^5}{1 - -3}$$

$$= \frac{12 - -2916}{4} = 732$$

Ex 7 Find sum
$3 - 6 + 12 \ldots$ to 7 terms

$n = 7, \; r = -2, \; a_1 = 3$

$$S_7 = \frac{3 - 3(-2)^7}{1 - -2}$$

$$= \frac{3 - -384}{3} = \boxed{129}$$