11-2: Arithmetic Series, part 1

The sum of the terms of an arithmetic sequence is called an arithmetic series.

ex.) Arith. sequence: $4, 7, 10, 13$ Arith Series: $4+7+10+13$

S_n represents the sum of the first n terms of a Series.

ex.) For $4+7+10+13$

$S_3 = 4+7+10 = 21$

$S_4 = 4+7+10+13 = 34$

For a large Series, it's unreasonable to add up a myriad of numbers. So, there must be a formula. Let's find it!!!

Ex.) $S_7 = 1+2+3+4+5+6+7$

To make adding easier, we can reverse the series order to get equal sum pairs.

$S_7 = 7+6+5+4+3+2+1$

ADD: $2S_7 = 8+8+8+8+8+8+8$

Twice the sum = 7 groups of 8

$$\frac{2}{2}S_7 = \frac{7(8)}{2}$$

$$S_7 = \frac{7(8)}{2}$$

In general, the pattern is:

$$\text{Sum} = \frac{\#\text{ of terms}(\text{first} + \text{last})}{\text{two}}$$

Arithmetic Series formula: $S_n = \frac{n}{2}(a_1 + a_n)$

* useful when you know n, a_1, a_n

Alternate version (substitute out a_n)

$$S_n = \frac{n}{2}\{a_1 + [a_1 + (n-1)d]\}$$

Simplify: $S_n = \frac{n}{2}[2a_1 + (n-1)d]$

Useful when you <u>don't</u> have a_n, but know d.

Hint: If you don't know n, must first find it using: $a_n = a_1 + (n-1)d$

(8) S_n when $a_1 = 4$, $a_n = 100$, $n = 25$

$$S_n = \frac{n}{2}(a_1 + a_n)$$

$$S_{25} = \frac{25}{2}(4 + 100)$$

$$= \frac{25}{2}(104) = 1300$$

$$\boxed{S_{25} = 1300}$$

(9) $a_1 = 40$, $n = 20$, $d = -3$

$$S_n = \frac{n}{2}[2a_1 + (n-1)d]$$

$$S_{20} = \frac{20}{2}[2(40) + (19)-3]$$

$$S_{20} = 10[80 + -57]$$

$$10[23] = \boxed{230}$$

(10) Find S_n when $a_1 = 132, d = -4, a_n = 52$

Find n, $a_n = a_1 + (n-1)d$

$52 = 132 + (n-1)-4$

$52 = 132 - 4n + 4$

$52 = 136 - 4n$

$-136 \quad -136$

$\frac{-84}{-4} = \frac{-4n}{-4}$

$n = 21$

$S_n = \frac{n}{2}(a_1 + a_n)$

$S_{21} = \frac{21}{2}(132 + 52)$

$= \frac{21}{2}(184) = \boxed{1,932}$

(11.) Find S_n if $d = 5$, $n = 16$, $a_n = 72$

Need a_1 ; so $a_n = a_1 + (n-1)d$

$72 = a_1 + (15)5$

$72 = a_1 + 75$

$-75 \quad -75$

$-3 = a_1$

$$S_n = \frac{n}{2}(a_1 + a_n)$$

$$S_{16} = \frac{16}{2}(-3 + 72)$$

$$S_{16} = 8(69) = 552$$

(12.) $5 + 11 + 17 + \ldots + 95$ Find S_n

Need n, so... $a_n = a_1 + (n-1)d$

$95 = 5 + (n-1)6$

$95 = 5 + 6n - 6$

$96 = 6n$

$n = 16$

$S_{16} = \frac{n}{2}(a_1 + a_n)$

$= \frac{16}{2}(5 + 95)$

$S_{16} = 800$