Ch 10, sec 1: Exponential Functions.

In an exponential function, the variable is now the exponent.

General form: $y = ab^x$
b is the base

ex.) $y = 2^x$, $y = -5 \cdot 2^x$, $y = 5^{-x}$
$y = -3^{2x}$

Exponential functions model population growth, spread of disease, decay, sound level, richter scale.

The Richter scale measures the size of an earthquake. The "number" we give it is expressed as 10^x.

# on Richter Scale	
1	$10^1 = 10$
2	$10^2 = 100$
3	$10^3 = 1{,}000$
4	$10^4 = 10{,}000$
5	$10^5 = 100{,}000$
6	$10^6 = 1{,}000{,}000$
7	$10^7 = 10{,}000{,}000$
8	$10^8 = 100{,}000{,}000$
9	$10^9 = 1{,}000{,}000{,}000$
10	$10^{10} = 10{,}000{,}000{,}000$

Compare 10^4 to 10^8.

$$10^4 \downarrow 10{,}000 \qquad 10^8 \downarrow 100{,}000{,}000$$

10^8 is 10,000 times Larger than 10^4.

Compare: $y = x^2$ $y = 2^x$

x	y
-2	4
0	0
5	25
10	100
50	2500
100	10,000

x	y
-2	.25
0	1
5	32
10	1024
50	1.12×10^{15}
100	1.26×10^{30}

$2^{-2} = \frac{1}{2^2}$

* Exponential functions start small, but grow larger and faster than polynomials.

Review: When bases are alike, and you multiply, you <u>add</u> exponents.

When dividing, subtract exponents.

$(2^a)^b$ = You multiply exponents

thus ... 2^{ab}

ex.) $7^{\sqrt{2}} \cdot 7^{\sqrt{3}} = 7^{(\sqrt{2}+\sqrt{3})}$

ex.) $\left(8^{\sqrt{3}}\right)^{\sqrt{5}} = 8^{\sqrt{15}}$

p. 600 #9.) $5^{1\sqrt{2}} \cdot 5^{3\sqrt{2}} = 5^{4\sqrt{2}}$

#10) $\left(3^{\sqrt{5}}\right)^{\sqrt{5}} = 3^{5} = 243$

#11.) $27^{\sqrt{5}} \div 3^{\sqrt{5}} =$

$\left(3^{3}\right)^{\sqrt{5}} \div 3^{\sqrt{5}}$

$3^{3\sqrt{5}} \div 3^{1\sqrt{5}} = \boxed{3^{2\sqrt{5}}}$

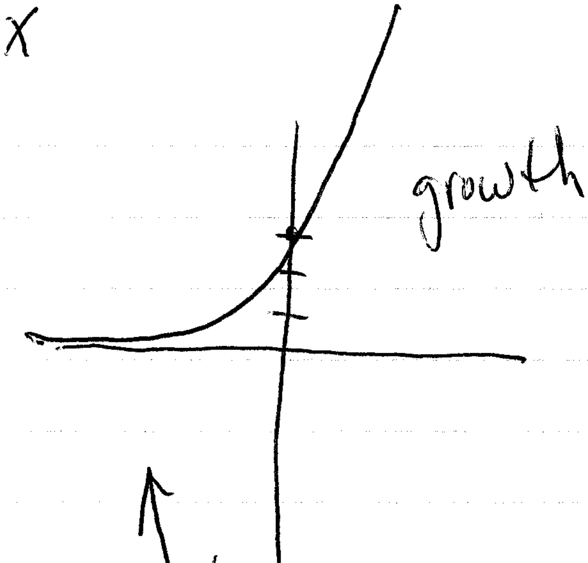
$9^{3} \cdot 81^{8} =$

$9^{3} \cdot \left(9^{2}\right)^{8}$

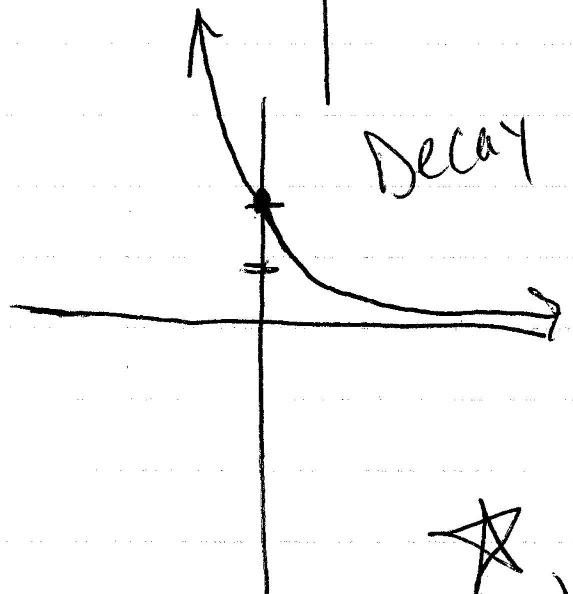
$9^{3} \cdot 9^{16} = \boxed{9^{19}}$

$$y = a\underline{b}^x$$

$$y = 3(2)^x$$



$$y = 2\left(\frac{1}{2}\right)^x$$



$y = a \cdot b^x$ contains $(0,1)$ & $(-1,3)$

$$\frac{3}{1} = \frac{\cancel{1} \cdot b^{-1}}{\cancel{1}}$$

$$\frac{1}{3} = b$$

$$a = 1$$

$$y = ab^x$$

$$y = 1\left(\frac{1}{3}\right)^x$$

$y = a(b)^x$ when $(\overset{x}{0}, \overset{a}{3})$ & $(\overset{x}{-1}, \overset{y}{6})$

Solve for b

$\frac{6}{3} = \frac{\cancel{3}(b)^{-1}}{\cancel{3}}$ $y = 3\left(\frac{1}{2}\right)^x$

$\frac{1}{2} = \frac{3}{6} = b$

$(0,3)$ & $(2,12)$ $y = a(b)^x$

$y = ab^x$ $\frac{12}{3} = \frac{3(b)^2}{3}$

$y = 3(2)^x$ $\sqrt{4} = \sqrt{b^2}$

$2 = b$

Simplify

Ex.) $(3^{\sqrt{3}})^{\sqrt{12}} \to 3^{\sqrt{36}} \to 3^{6} \to 729$

Ex.) $(x^{\sqrt{3}})^{\sqrt{8}} \to x^{\cancel{\sqrt{24}} \to \sqrt{4}\sqrt{6}} \to x^{2\sqrt{6}}$

Ex.) $5^{\sqrt{2}} \cdot 5^{\cancel{\sqrt{18}} \to \sqrt{9}\sqrt{2}} =$

$5^{\sqrt{2}} \cdot 5^{3\sqrt{2}} = 5^{4\sqrt{2}}$

Ex.) $x^{3\pi} \div x^{\pi} = x^{2\pi}$

Ex.) $3^{x} > 9$ $\qquad x > 2$

Like Bases

$\cancel{3}^{x} > \cancel{3}^{2}$

Drop Base $x > 2$

Ex.) $2^{2x+3} = 32$

$$2^{2x+3} = 2^5$$

$$2x + 3 = 5$$
$$\quad -3 \quad -3$$

$$2x = 2$$

$$\boxed{x = 1}$$

Ex.) $49^x \leq \frac{1}{7}$

$$7^{2x} \leq 7^{-1}$$

$$\frac{\cancel{2}x}{\cancel{2}} \leq \frac{-1}{2}$$

$$x \leq -\tfrac{1}{2}$$

Ex.) $27^{x} = 9^{2x+3}$

$\downarrow \qquad \downarrow$

$3^{3(x)} = 3^{2(2x+3)}$

$$3x = 4x + 6$$
$$-4x \quad -4x$$

$$\frac{-x}{-1} = \frac{6}{-1}$$

$$\boxed{x = -6}$$